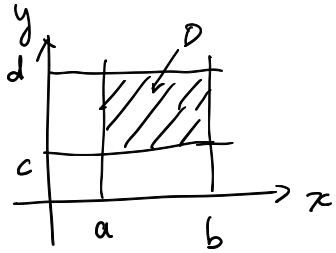
MATH 2E Prep: Double integrals

Facts to Know:

- If $D = \{(x, y) \mid a \leqslant x \leqslant b,\ c \leqslant y \leqslant d\}$, then $\iint_D f(x,y)dS = \int_c^d \int_a^b f(x,y)\,dx\,dy$

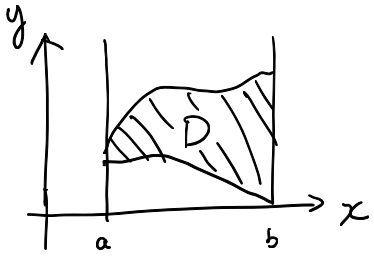
  $= \int_a^b \int_c^d f(x,y)\,dy\,dx$

  Sketch of D:



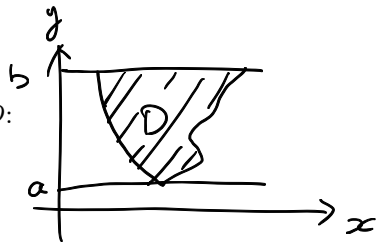
- If $D = \{(x, y) \mid a \leqslant x \leqslant b,\ g(x) \leqslant y \leqslant h(x)\}$, then $\iint_D f(x,y)dS = \int_a^b \int_{g(x)}^{h(x)} f(x,y)\,dy\,dx$

  Sketch of D:



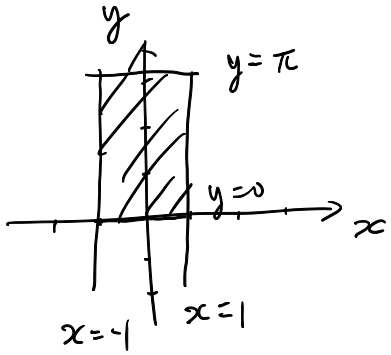
- If $D = \{(x, y) \mid a \leqslant y \leqslant b,\ g(y) \leqslant x \leqslant h(y)\}$, then $\iint_D f(x,y)dS = \int_a^b \int_{g(y)}^{h(y)} f(x,y)\,dx\,dy$

  Sketch of D:



Examples:

1. Evaluate $\iint_D x^2 \sin(y) dS$, where D is the rectangle bounded by lines $x = -1$, $x = 1$, $y = 0$ and $y = \pi$.



$$\iint_D x^2 \sin(y)\, dS = \int_{-1}^{1} \int_0^{\pi} x^2 \sin(y)\, dy\, dx$$

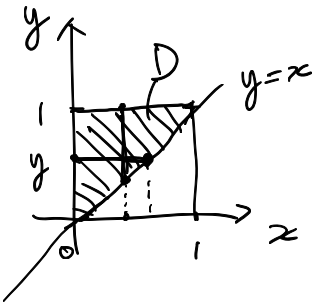
$$= \int_{-1}^{1} \left[-x^2 \cos(y) \right]_{y=0}^{\pi} dx$$

$$= \int_{-1}^{1} \left[-x^2(-1) - x^2(1) \right] dx$$

$$= \int_{-1}^{1} 2x^2 dx = \frac{2}{3}x^3 \Big]_{-1}^{1} = \frac{2}{3} \cdot 1 - \left(\frac{2}{3}\right)(-1) = \frac{4}{3}$$

2. Evaluate the iterated integral $\int_0^1 \int_x^1 e^{y^2} dy dx$.

$\int e^{y^2} dy = ???$ not work.



$$\int_0^1 \int_x^1 e^{y^2} dy dx = \iint_D e^{y^2} dS$$

$$= \int_0^1 \int_0^y e^{y^2} dx dy$$

$$= \int_0^1 \left[x e^{y^2} \right]_{x=0}^{x=y} dy$$

$$= \int_0^1 y e^{y^2} dy$$

$$= \frac{1}{2} \int_0^1 e^{y^2} 2y dy$$

$$= \frac{1}{2} \int_{y=0}^{1} e^u du$$

$$= \frac{1}{2} \int_{u=0}^{1} e^u du = \frac{1}{2} e^u \Big]_{u=0}^{u=1}$$

$$= \frac{1}{2}(e-1)$$

$u = y^2$,

$u' = \frac{du}{dy} = 2y$

$\Rightarrow du = 2y dy$